Review in development of choatic mixing

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Abstract

This study was conducted to draw and develop a general framework for the development of chaotic mixing. The importance of mixing and feedback of variety of flows were showed. The key concepts were introduced along with some examples and main applications of chaotic mixing .

Keywords mixing of flows, chaotic advection, stirring of fluids, bounded flows.

INTRODUCTION

Chaotic mixing has been receiving an increasing interest not only because of its various application areas, but also because it presents an approach that can lead us to understand various kinds of dynamical phenomena occurring in different systems such as physical, biological and environmental flows. Furthermore, chaotic advection has been widely used by researchers in diverse areas of fluid mechanics to explain a variety of experimental or real world phenomena Aref,2002, 13](Ottino, 2010).

History of chaotic mixing

The work on advection in two-dimensional time dependent flows and on advection in steady three-dimensional flows is related to the development of the novel phenomenon of the mixing process called Chaotic Advection by Hassan Aref in the early 1980s .

"The term `chaotic advection' was first introduced in the title of an abstract for the 35th annual meeting of the American Physical Society (APS) Division of Fluid Dynamics (DFD) in 1982. The main reference, a Journal of Fluid Mechanics paper published in 1984, may be the true `birth date' of the term (Aref,2002)

Furthermore, the combined NATO and European Geophysical Society workshop on Chaotic Advection, Tracer Dynamics and Turbulent Dispersion held in Italy in 1993 was the first major meeting to have "Chaotic Advection" in the title. In fact, the first appearance of the two words `Stirring' and `Mixing' was in a paper for Carl Eckart in journal of Marine Research to distinguish the "mechanical" and "molecular" physical processes that produce mixing (Aref,2002). In that, Eckart wrote:

" the effect of advection is appropriately called stirring" and "the effect of conduction on or diffusion is to decrease the mean value of the gradients. This is appropriately called mixing. Ordinarily, the early stages of the process... will be dominated by the advective processes... These may so increase the mean gradient that the mixing process will ultimately dominate... Viscosity, if not counteracted by other factors, ends to stop the stirring... before an appreciable amount of mixing can be occurred" .

Today, the name of Carl Eckart is associated with the Wigner-Eckart theorem in mechanics. However, chaotic advection has also been named by different terms:`Lagrangian turbulence' (Chaiken et al., 1987and Dombre et al., 1986), `Chaotic mixing' (Chien et al.,, 2003) and `Chaotic convection' (Ott and Antonsen, 1988). In fact, earlierwork on advection has been done early in the 1960s by Arnol'd and Hénon on steady three-dimensional flows. This work -unfortunately - had not been widely appreciated despite related ideas and results it had contained and consequently" Chaotic Advection" had to wait another twenty years to appea(Aref, 1984). This was the first time that this term has been used as a scientific concept and one of the key words of papers and journals related to chaotic mixing. We conclude this section with this sentence quoted from(Aref,2002) 'Essentially, what is being proposed is the existence of a new advective regime,intermediate between turbulent and laminar advection, which one might call `Chaotic Advection'.

MATERIALS

Practical examples of chaotic mixing
1-Blinking vortex (BV) flow:

In his early work on advection by interacting point vortices, Aref (1984) introduced the blinking vortex BV flow. This model is mathematically described by a flow in a circular boundary of radius a. The system consists of two fixed point vortices separated by a fixed distance 2b. The agitating vortex with the position Z(t) is generally given as a function of time t,

$$Z(t) = b\, f(t/T) \qquad (1)$$

where b and T are constants with an amplitude b < a and f is a real periodic function with period unity. The vortex strength of the agitator is given by Γ and in this case the strength of its image (outside the circular boundary) will be given by −Γ. Both, the agitator and its image provides a system of unsteady flow. This idealised model is completely parametrised by two quantities, β which gives the dimensionless amplitude for the location of the agitator

$$\beta = \frac{b}{a} \qquad (2)$$

and μ which is the period of its motion,

$$\mu = \frac{\Gamma T}{2\pi a^2} \qquad (3)$$

The agitator is assumed to be `on' when the corresponding point in the flow domain works as a point vortex. Under a single stirrer if a particle moves very close to one of the vortices, its motion draws a circle in the flow that dominates around the point. This flow is produced by this point vortex modulo the slip boundary condition on the bounding circle. The flows stops immediately when the agitator is `turned off'. Through this system, an incompressible two-dimensional inviscid unsteady flow has been considered. In order to demonstrate chaotic behaviour, this model of flows has been studied through Several simulations.

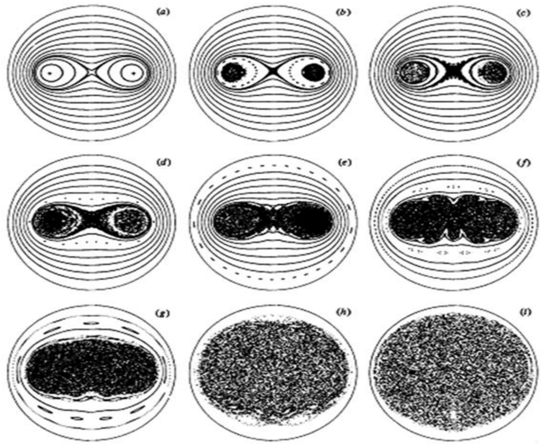


Figure 1: Poincare section of particle positions showing motion regular to chaotic. Cross indicate agitator positions (Aref, 1984).

The parameters considered in this protocol are Γ = 2π and a = 1 so that β = b and μ = T in (2) and (3), respectively. These parameters, β and μ control the efficiency of the stirring protocol. The time interval was taken to equal the period of Z(t) in (1). Several sample particle trajectories according to the parameters β = 0.5 and μ = 1.5 or 0.5 were tested. As seen in these tests, the two point vortices blink `on' and `off' periodically. A length of circular arc is drawn around each agitator and when the agitator is `on', the particle moves around it in a family of circles giving a description of its motion. Figure 1 shows some numerical results obtained according to this model of flows.For all the cases shown in this figure β = 0.5 and the apparent results differ from each other by varying the time interval μ(= T).

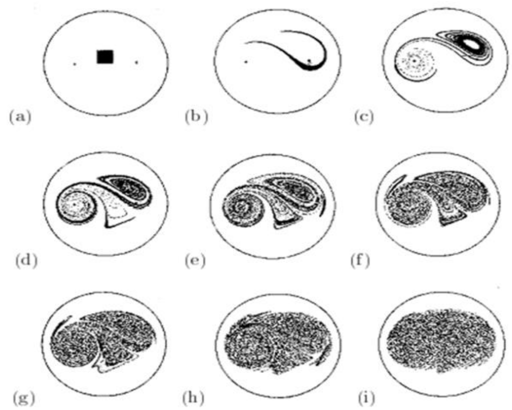


Figure 2 : Phases in the stirring of an initially a square array of particles (Aref, 1984)

This figure shows regular formulations of curves presented by two vortex agitators fixed at Z = 1/2 and Z = −1/2.
The streamlines of the particle motion can be seen in panel(a) of this figure whereμ = T is short and so is approximately small as two agitators being `on' simultaneously. The regular formations vanish gradually as μ is increased, turning the behaviour to a chaotic motion which sets in with μ = 0.1 (panel (b)). This chaotic component of the phase space increases as μ increases to 1.5 as seen in panel (i) where no regularity of curves can appear except may be near the boundaries. This actually leads to an efficient stirring where particles may be found everywhere inthe chaotic region of the flow. Figure 2 shows different phases in the stirring of an initially square array marked by 10000 particles with parameters β = 0.5 and μ = 1.0. These plots are obtained at times started with t = 0 as in panel(a) where mixing has been marked by dark and white flow. As we can see, a complicated behaviour of a scalar is observed through the phase spaces leading to a formulation of fine scales shown in this figure. However, the blinking vortex flow is considered the simplest possible model of flow in which chaotic advection can be found. The motivation for this model relates to a study by Aref & Pomphrey (1980) on interacting point vortices, where four point

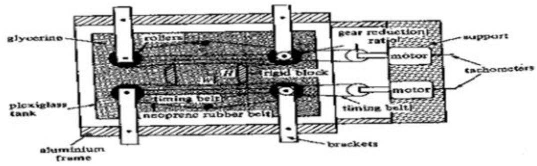


Figure 3: Flow region. The dimensions of the cavity are width W = 10.3 cm and height H = 6.2 cm (Ottino, 2010).

vortices were considered to produce chaotic advection (Aref and Pomphrey, 1990).
Furthermore, the `blinking-rotlet flow' studied by Meleshko & Aref (1996) is considered as an extension to this flow (Meleshko and Aref, 1996).

2- Mixing of fluorescent dye in glycerine:

Most experiments on mixing in flows were carried out using cavity flows. These experiments are conducted to study several classes of steady and time-periodic flow. Most interesting are experiments of mixing in two-dimensional flows conducted chemically by Ottino (Ottino, 2010), where mixing was displayed visually by a fluorescent dye dissolved in glycerine This system consists of a rectangular region capable of producing a two-dimensional velocity field in the x−y plane. This experimental system is based on an improved system (Fig. 3) described by Chien, Rising & Ottino (1986). This system consists (as shown in Fig. 3) of two sets of roller wheels connected by belts driven separately by motors and two bands acting a moving walls (Chien et al.,, 2003).
Entirely, this system is sunk in a tank of one foot depth filled with glycerine. This tank has been chosen suitably to visualise mixing. The size of the region of the flow can be changed to a maximum area 5 × 5.5 inches. In these experiments, the velocity of the top and bottom walls is denoted by v_{top} and v_{bot}, respectively. These velocities are generally functions in time and specified using a computer control system. The first experiment was conducted using steady flows. Here, the velocities of the top and bottom walls are v_{top} = v_{bot}= 1.58 cm/s with Reynolds number Re = 1. This experiment shows poor mixing resulting by a tracer (fluorescent dye) placed

vertically in a steady flow (glycerine). The stretching of the flow clearly can be seen where the motion is considered to be integrable as the flow is steady. Both walls in some parts of this experiment are moved in the same direction with velocities v_{top} = v_{bot} = 1.58 cm/s, while in some others the walls are moved in opposite direction with v_{top} = $-v_{bot}$ = 1.58 cm/s and in the final part, just the top wall
has been moved with v_{top} = 1.58 cm/s and v_{bot}=0. (For more details see (Chien et al.,, 2003)).

Mixing in global chaotic flows

Even though the first theoretical example of chaotic advection was a three dimensional Flow (Hénon, 1966), the theoretical studies addressing chaotic mixing in three dimensions are still few. However, mixing has been studied in idealised models: in deterministic or random flows. One of these studies includes the decay phase of a scalar field undergoing steady three-dimensional chaotic advection by Toussaint et al. (2000). This research showed that the spectral decay is found to be exponential and self-similar at large times, with the focus on the importance of the asymptotic decay-time as a measure of the efficiency of mixing. The power law variance spectrum in this study is found at scales intermediate between the large and the smallest ones, at which diffusion is effective. These flows exhibit global chaos, so that no integrable islands are present, the decay of a passive scalar is exponential,at a rate γ(κ) that tends to a constant independent of the diffusivity as κ → 0 (Toussaint et al., 2009).

Mixing in flows with boundaries

Several predictions for the peripheral mixing have been recently made (ChertkovyandLebedev,2003;Lebedevan Turitsyn, 2004) and checked with laboratory experiments in a chaotic micro-channel (Simmonet and Groisman, 2005), in polymer solution [5-6] and in kinematic simulations (Salman and Haynes, 2007). By `peripheral' we mean the region of the flow near the wall boundaries where the slip (n ·u = 0) and no-slip (u = 0) boundary conditions are considered together with a flow of poor mixing. Modern steps in the understanding of the passive scalar transport recently have been made by Chertkov and Lebedev (2003a), Lebedev & Turitsyn (2004), and Chernykh & Lebedev (2008). These authors made the key observation that near to a no-slip boundary the effective time scale of mixing becomes very long and the resulting boundary layer can control the mixing of passive scalar by acting as a reservoir, slowly releasing the scalar into the body of the flow, where mixing occurs more rapidly. If the flow is random, then near the boundary the effective correlation time of the flow becomes very short (as the turnover time increases). Approximation leads to a diffusion equation determining the boundary layer structure and giving predictions about decay rates and scalar field moments. This has been further explored in (Salman and Haynes, 2007) by Salman & Haynes who use families of `renewing' flows, in which the random flow is chosen to be independent and identically distributed on time intervals [jτ, (j+1)τ]. Specifically, the focus of the above studies for closed systems is the decay of passive scalar fluctuations for flows obeying a no-slip condition, where the scalar obeys a no-flux or Neumann condition, in a plane layer domain with periodic boundary conditions along the layer. In this case the long-time regime is exponential decay with a decay rate γ(κ) proportional to κ 1/2. These authors identified different distinct regimes including a transient algebraic decay.

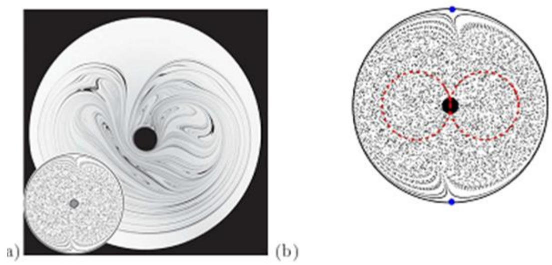


Figure 4. (a) Chaotic mixing experiment in a closed vessel. The figure-eight stirring protocol . (b) Poincare section obtained

numerically for the corresponding flow (Gouillart et al., 2007).

Such a scenario was first identified theoretically by Chertkov & Lebedev (2003a) followed
by Lebedev & Turitsyn (2004) and later examined by Salman & Haynes (2007). (see (Chertkovy and Lebedev, 2003)(Lebedev and Turitsyn, 2004) and (Salman and Haynes, 2007). In particular, these theoretical predictions suggest four stages with an intermediate regime in which the scalar variance evolves according to a power law. The slip casein this scenario is described by two stages,

• First stage: the scalar variance remains constant producing thin structures of fluctuations (Fig. 6(a,c)). Once the thickness of these structures decreases enough (Fig 6(e)), diffusion smears out high gradients in the scalar field.

• Second stage: followed rapid mixing in the interior of the flow, a long-time behaviour is observed leading to an exponential rapid decay of the scalar variance (Fig. 6(g,i)). In contrast, the no-slip case is characterised by four separate stages,

• First stage: the intense stirring of the scalar field is produced within the interior of the flow where diffusion is not important (Fig 6(b)).

• Second stage: after rapid mixing of the scalar in the bulk of the fluid, there is an intermediate regime of mixing where diffusion becomes important as seen in (Fig. 6(d,f)). The slow release of tongues (Fig. 6(f,j)) of relatively large scale scalar from the boundaries leads to a slow `algebraic' decay of concentration variance in this stage discussed from the point of view of experiment and theory (Gouillart et al., 2008), (Gouillart et al., 2007) and (Gouillart et al., 2010).

• Third stage: the scalar becomes well-mixed in the body of the flow, but remains slower in a region near the boundaries that decreases with time.

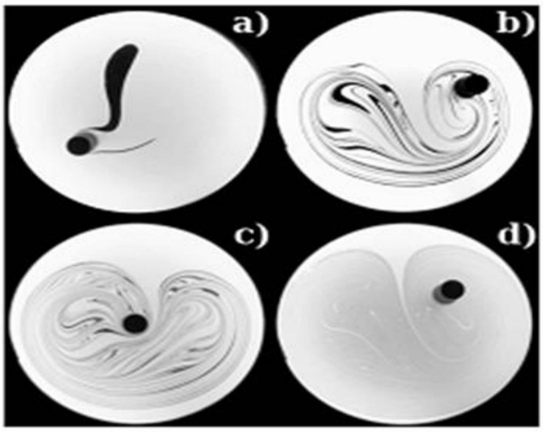


Figure 5. Successive stages of homogenisation for a blob of dye stirred by the figure-eight protocol (Gouillart et al., 2008).

• Fourth stage: once the boundary layer width becomes of the order of diffusion boundary layer width, the decreasing stops and the scalar trapped near the walls controls the decay rate, turning the `algebraic' decay to an exponential one; V ' exp(−2γt) (Fig. 6(j)).

In (Lebedev and Turitsyn, 2004), the theory of Lebedev & Turitsyn (2004) has been supported by numerical simulations of scalar transport in three-dimensional Couette flow by Boetta, De Lillo
& Mazzino (2009) (see Boffetta et al., 2009) and extended to the case of turbulent flow, where the wall layer has a more complex structure (Garcia and barra, 2009) (Skvortsov and Yee, 2011).

This is for flows in closed systems, bounded regions of the plane or a plane layer with periodic boundary conditions. In their study, Salman & Haynes (2007)emphasised the importance of the existence of the no-slip boundaries on the decay rate. Close to such boundaries the velocity field of the flow tends to zero and the advection slows down in the peripheral regions while mixing continues to be efficient in the bulk of the flow away from the boundary. Experimentally, most interesting are observations of the mixing dynamics in two dimensions, conducted with numerical simulations in closed vessel using fully chaotic flows by Gouillart et al. (2007, 2008); Gouillart, Thieault & Dauchot (2010). Gouillart et al. (2007) investigated dye homogenisation conducted in a closed vessel, where a single rod has gently been stirred the fluid with a figure of eight motion (see Fig. 4). Such mixers showed an important effect of 'slow' algebraic decay imposed by no-slip walls. The no-slip boundary condition at the walls of the vessel can strongly influence the decay of the scalar. The existence of parabolic points on such boundaries (as a result of the hydrodynamics of the no-slip walls) considerably shows down

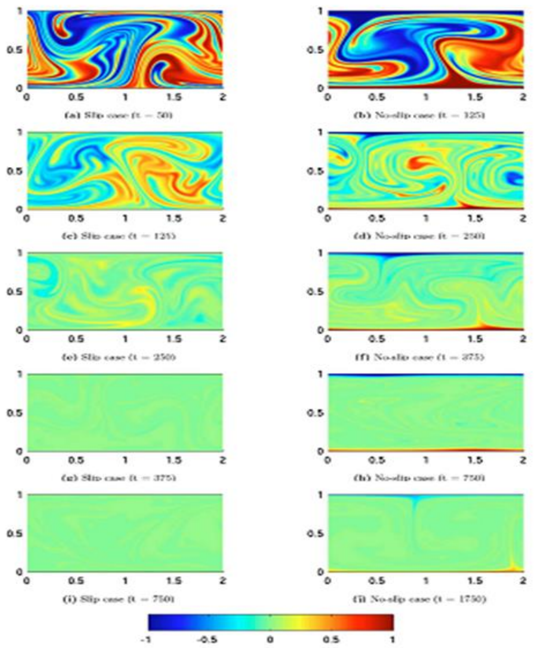


Figure 6. Scalar distribution of a random flow in a plane layer with slip boundary condition (left column) and no-slip boundaries (right column) with Neumann boundary conditions at t = (a) 50, (b, c) 125, (d, e) 250, (f, g) 375, (h, i) 750, and (j) 1750 (Salman and Haynes, 2007).

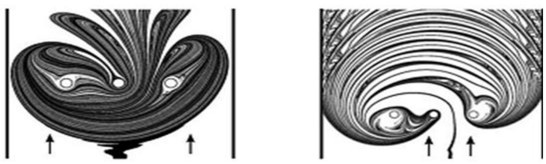


Figure 7. Numerical simulations of a flow in an open channel with a figure-eight rod stirring protocol (Thiffeault et al., 2008).

mixing, turning an exponential decay into a power law [15-17]. Furthermore, the distinct stages of mixing successfully have been examined in a closed system by Gouillart et al. (2008) (see Fig. 5). In fact, three stages of mixing have been observed and can be linked to the four stages discussed earlier in this section. These stages show the homogenisation for a blob of dye stirred by the figure-eight protocol. We can see the boundary layer thinning in Fig. 5(c,d). These experimental (Burghelea et al., 2004; Simmonet and Groisman, 2005; numerical; Chertkovy and Lebedev, 2003; Lebedev and Turitsyn, 2004; Salman and Haynes, 2007; Chernykh and Lebedev 1987) studies obtained an exponential decay for the scalar field in a chaotic mixer at large times. It has been observed that this behaviour is attributed to convergence the scalar field to the `strange' eigenmode discussed by Pierrehumbert & Yang (1993); Gouillart et al. (2009) and later by El Omari & Le Gu (2010). In spite of the importance of the Dirichlet case in linking questions of chaotic advection to heat transfer, and possible applications, it has been little studied. However, the recent numerical work of El Omari & Le Guer (2010) considers decay of temperature within a two-rod mixing device under various protocols for rotating the rods. The full Navier-Stokes equation for the fluid flow is solved. The main result of this work is that the heat transfer is controlled by boundary layers on the no-slip boundaries. Several other studies have used flows with 2π-periodicity, but with a focus on the presence of islands and other structures in the fluid flow, rather than of boundary conditions for the fluid flow and scalar (Cerbelli et al., 2003)[29].

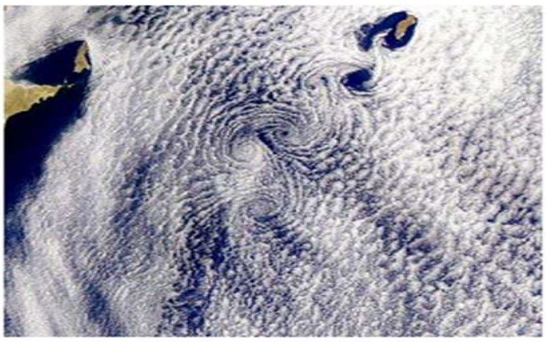
Mixing in open flows

Chaotic advection in open hydro-ynamical flows is a ubiquitous phenomenon. Undergoing chaotic advection, mixing has been investigated experimentally in open systems. A further element of the study (Lebedevand Turitsyn, 2004) concerns the decay of the scalar variance in the open system consisting of a flow down a pipe under the influence of no-slip boundary conditions;

Figure 8: Stretching and folding of clouds in the wake of Guadalupe island. From the NASA archive.

this geometry allows experimental verification of theory, and is particularly important in mixing applications for microfluidics (Simmonet Groisman, 2005 and Burghelea et al., 2004; Burghelea et al., 2004). For example, the experiment of Burghelea (Boffetta et al., 2009 and Burghelea et al., 2004 was carried out in a micro-channel using viscous fluid with polymer. In this experiment, it has been verified that following an initial rapid mixing of the scalar concentration, a decreasing rate was observed close to the boundaries.

Moreover, the scaling of mixing length with Péclet number derived by Chertkov &Lebedev (2003a) and Lebedev & Turitsyn (2004) in the intermediate regime were also observed in this work. Numerical simulations of open flow were done also by Thieault et al.(2008) and Gouillart et al.(2009) as a subsequent work to Gouillartet al.(2007). As mixing can greatly be enhanced by stirring a fluid, Thieault et al.(2008) considered a figure- eight rod stirring protocol in an open channel shown in Fig.7. This protocol is capable of producing intense stretching that is important for chaotic mixing. In the second paper (Gouillart et al., 2009),two different stirring protocols are studied; while the mixing region is separated from the walls in the first one, it extends to the walls in the second. This study provided an experimental evidence for convergence of the scalar to the strange eigenmode that is previously discussed by Pierrehumbert & Yang (1993) in a closed system.



Besides, this research emphasised on the importance of no-slip boundaries in producing non-self similar mixing dynamics. Numerically and experimentally, these works obtained an exponential decay rate for the variance of the scalar in chaotic mixers with the focus the important role played by the no-slip boundaries on the decay of the scalar variance.

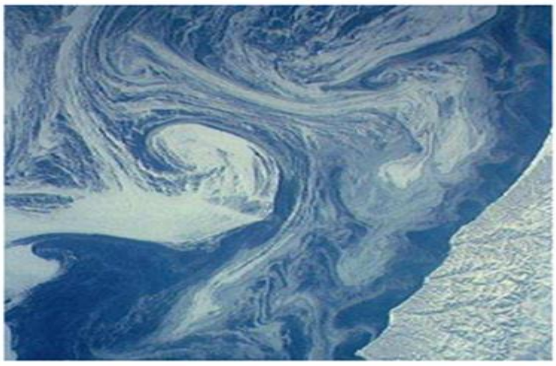


Figure 9: Sea – ice distribution close to Kamchatka. From the NASA archive. (http: // images. jsc.nasa. gov /imas /images /earth /STS045 /html /10068879.htm).

Although the experimental results provided the conformation of some scaling laws derived by Chertkov & Lebedev (2003a); Lebedev & Turitsyn (2004) and Chernykh & Lebedev (2008) they do not fully and exclusively test other elements of theory. Furthermore, the slowly evolving stage of mixing implies that a very long time is required to realise the final stage of mixing. In the experiment of Burghlea et al., (2004) this means that a very long channel is required making it difficult to realise the full behaviour of mixing.

Main applications:

understanding such mixing and enhanced diffusion processes in deterministic, random and turbulent flows is key in many important applications such as:

• Stirring of fluids on geophysical or planetary scales. Due to the convective motions in the Earth's mantle, numerical simulations show signs of chaotic advection represented by folds and layers on smaller and smaller scales. This can be seen in individual rocks (Aref, 2002and Ottino, 2010).

•In the field of medicine where low Reynolds number mixing enabled human

plasma mixing without damaging the cells.
•Enhanced heat transfer for heat exchange (El Omari and Le Guer, 2010).
•Stirring and mixing devices in chemical engineering process combustion (Ottino, 2010).
• Mixing and transport phenomena in biological systems (e.g., Pulsatile flow in corrugate channels and flow in twisted tubes).
•Acceleration of chemical reactions.
.•In the atmosphere, where the flow is dominated by an azimuthal jet, such as might be encountered in the polar night jet surrounding the ozone hole in Antarctica (Aref,2002).
•In environmental flows; in the atmosphere (Fig. 8) and in the ocean (Fig. 9) as plotted by NASA. The former shows a flow creating folds in cloud formations, formed by the wind in the wake of Guadalupe island. The latter shows the flow close to Kamchatka, where the stretching and folding of the flow that occur in the sea can be seen.
The list of applications continues to grow.

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